Reduced Order Modeling of Transcritical AC System Dynamics Using Singular Perturbation

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ABSTRACT

This paper presents a reduced order dynamic model of a transcritical air-conditioning system, specifically suited for multivariable controller design. An 11^{th} order nonlinear dynamic model of the system is derived using first principles. Two methods of deriving the governing equations are presented. The first method simplifies the governing partial differential equations using lumped parameter assumptions. The second method uses the unsteady state conservation equations, and is shown to be equivalent to the first method. Analysis of the resulting model indicates that the system is singularly perturbed. The model reduction procedure exposes that the first derivation approach results in a model ill-suited for model reduction. The second modeling approach is shown to be simpler conceptually, and well suited for model reduction. The model reduction procedure yields physical insight as to which physical phenomenon are relatively fast/slow, as well as providing a 5^{th} order dynamic model appropriate for multivariable controller design. Although all results shown are for a transcritical cycle, the methodology presented can easily be extended to the more common subcritical cycles.

1 INTRODUCTION

An increasing amount of research is devoted to the development of control-oriented models of vapor compression cycles [2,3,5,8,9]. These systems are used for refrigeration, air-conditioning, and heating, and have traditionally been controlled using simple Single-Input Single-Output (SISO) techniques. However, research has demonstrated that this approach has difficulties, and results in performance inferior to advanced control strategies [3]. Developing advanced control strategies applicable to various types of air-conditioning and refrigeration systems requires a control-oriented model that is accurate enough to be useful and simple enough to be practical. While the actuating components (compressor, expansion valve, fans) are often modeled with simple algebraic relationships, developing a simple yet accurate model of the complex two-phase flow dynamics of the heat exchangers is a formidable task. A common method for modeling these dynamics is using a lumped parameter moving boundary approach. This method assumes that the heat exchangers are divided into several regions defined by the fluid phase (liquid, two-phase, vapor). Average properties are assumed for the single-phase regions, and a mean void fraction assumption is used to characterize the two-phase flow region [1]. The boundaries between regions are allowed to be dynamic to capture essential transient behavior.

The resulting models developed using this approach are significantly simpler than those developed using finite difference techniques, but are still more complex than desired for the application of advanced control strategies. Recent research has shown that although the resulting models are generally 10^{th}-12^{th} order, empirical models require only 3^{rd} order models to capture individual input-output behavior [8]. Furthermore, the analysis of the linearized dynamic models indicates that several dynamic modes are significantly faster than the dominant system dynamics. Although numerical model reduction techniques could be used to develop reduced order models, this approach yields no physical insight into which dynamic modes are relatively fast/slow. Singular perturbation techniques justify model reduction while preserving the physical nature of the dynamic states. This method is applied to the dynamic model of a transcritical air-conditioning cycle, with the objective of developing a low order control-oriented model of the system, while preserving the physical meaning of the state variables.

This paper builds upon the previous work of Rasmussen et al [8]. Although much of the material presented in [8] is necessary background for these research results, space constraints allow only a short summary of the results to be included here. The interested reader is encouraged to refer to [8] for more information.

The remainder of this paper is organized as follows. Section 2 gives a qualitative description of the system considered, and presents a method for deriving the governing dynamic equations from the governing PDEs. Section 3 presents an alternative modeling procedure more suitable for model reduction. Section 4 presents and analyzes the linearized versions of the derived models. Section 5 presents an empirical MIMO model constructed using sub-space system identification techniques. Section 6 details the application of singular perturbation techniques and the resulting reduced order model. The nonlinear, linear, and reduced order models are compared to experimental data in Section 7. Concluding remarks and an outline for future work are given in Section 8.

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2 MODELING APPROACH #1

Recently, more attention has been given to replacing traditional automotive vapor compression cycle refrigerants with carbon dioxide. The resulting vapor compression cycle is uniquely characterized by the supercritical state of the refrigerant in the gas cooler. Generally an internal heat exchanger is included to increase capacity, which also increases the inherently coupled nature of the dynamics, and ensures that only refrigerant vapor enters the compressor (Figure 1).

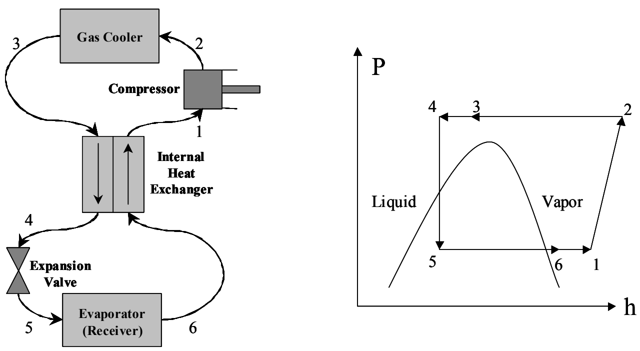

Figure 1: Diagram of Transcritical Vapor Compression Cycle

The four controllable inputs to this system are assumed to be compressor speed, expansion valve opening, and mass flow rates of air across the evaporator and gas cooler. The outputs of interest are superheat temperature (measure of efficiency), evaporator outlet air temperature (measure of comfort), as well as the operating pressures in the evaporator and gas cooler.

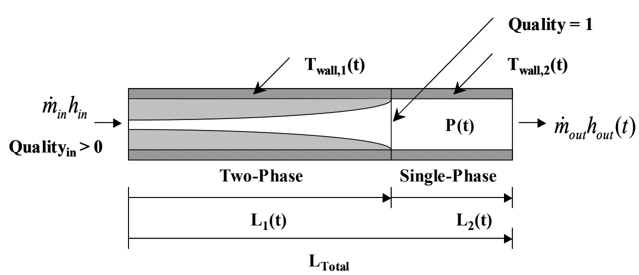

Figure 2: Evaporator with Two-phase Flow at Entrance and Superheated Vapor at Exit

Discussion regarding the derivation of the component models using the governing PDEs is included in [8] with detailed derivations presented in [7]. In summary, each of the five components is modeled separately and then combined to form the overall system model. The compressor and expansion valve are assumed to act instantaneously and are modeled with algebraic relationships. The internal heat exchanger is modeled using lumped capacitance assumptions, and results in a 3rd order model. The derivation procedure for the evaporator and gas cooler requires the integration of the governing PDEs (Equations 1-3) along the length of the heat exchanger to remove spatial dependence using the integration rule given in Equation 4. This approach can be tedious and requires a significant amount of algebraic manipulation. This approach has been used previously to model subcritical system components in [3,5,6,11]. Assuming two separate fluid regions (Figure 2) the evaporator is modeled as a 5th order system. The procedure for the gas cooler is similar resulting in a 3rd order model. Combining the 5 component models results in an 11th order nonlinear model of a transcritical a/c system.

$$\frac{\partial \rho A}{\partial t} + \frac{\partial \dot{m}}{\partial z} = 0 \tag{1}$$

$$\frac{\partial(\rho A h - AP)}{\partial t} + \frac{\partial(\dot{m}h)}{\partial z} = \pi D_i \alpha_i (T_w - T_r) \tag{2}$$

$$(C_p \rho A)_w \frac{\partial T_w}{\partial t} = \pi D_i \alpha_i (T_r - T_w) + \pi D_o \alpha_o (T_a - T_w) \tag{3}$$

$$\int_{z_1(t)}^{z_2(t)} \frac{\partial f(z,t)}{\partial t} dz = \frac{d}{dt}\left[\int_{z_1(t)}^{z_2(t)} f(z,t)\,dz\right] - f(z_2(t),t)\frac{d(z_2(t))}{dt} + f(z_1(t),t)\frac{d(z_1(t))}{dt} \tag{4}$$

For discussion purposes, the final form of the gas cooler model is given in Equations 5-7, with the states and input vectors defined in Equations 8 and 9 respectively. This component is the simplest case for modeling heat exchangers, and will be used to compare modeling approaches.

$$Z(x,u) \cdot \dot{x} = f(x,u) \tag{5}$$

$$f(x,u) = \begin{bmatrix} \dot{m}_{in}h_{in} - \dot{m}_{out}h_{out} - \alpha_i A_i (T_r - T_w) \\ \dot{m}_{in} - \dot{m}_{out} \\ \alpha_i A_i (T_r - T_w) - \alpha_o A_o (T_w - T_a) \end{bmatrix} \tag{6}$$

$$Z(x,u) = \begin{bmatrix} \left[\left(\left.\frac{\partial \rho_c}{\partial P}\right|_{h_c}\right) h_c - 1\right] A_{cs} L_T & \left[\left(\left.\frac{\partial \rho_c}{\partial h_c}\right|_{P_c}\right) h_c + \rho_c\right] A_{cs} L_T & 0 \\ \left(\left.\frac{\partial \rho_c}{\partial P}\right|_{h_c}\right) A_{cs} L_T & \left(\left.\frac{\partial \rho_c}{\partial h_c}\right|_{P_c}\right) A_{cs} L_T & 0 \\ 0 & 0 & (C_p \rho V)_w \end{bmatrix} \tag{7}$$

$$x = \begin{bmatrix} P & h_c & T_w \end{bmatrix}^T \tag{8}$$

$$u = \begin{bmatrix} \dot{m}_{in} & \dot{m}_{out} & h_{in} & T_{air,in} & \dot{m}_{air} \end{bmatrix}^T \tag{9}$$

3 MODELING APPROACH #2

Instead of starting with the governing PDEs and applying simplifying assumptions, an alternative approach is to use the unsteady-state equations for conservation of mass and energy, and assume control volumes associated with each region of refrigerant and wall. The resulting equations can be expanded into the desired form with freedom in choosing the state variables. To illustrate this approach, the derivation for the gas cooler is presented. Two alternative choices for state variables will be presented and will be denoted as x' and x'' (the original state representation is denoted as x). All representations can be shown to be equivalent.

The conservation of refrigerant energy for each region is given in Equation 10 where $\dot{U}$ is the rate of change of the total internal energy of the refrigerant in the region considered, $\dot{H}_{in}$ is the rate of energy entering the region by means of refrigerant mass, $\dot{H}_{out}$ is the rate of energy leaving the region by means of refrigerant mass, $\dot{Q}_w$ is the rate of energy leaving the region through heat transfer to the heat exchanger wall, and $\dot{W}$ is the work done by fluid (both moving boundary work and flow work) and is defined as $\dot{W} = \frac{d}{dt}(PV)$. Note that defining enthalpy as $H = U + PV$, Equation 10 can be simplified to Equation 11 where $\dot{H}$ is the rate of change of the total energy of the refrigerant.

$$\dot{U} = \dot{H}_{in} - \dot{H}_{out} - \dot{Q}_w - \dot{W} \tag{10}$$

$$\dot{H} = \dot{H}_{in} - \dot{H}_{out} - \dot{Q}_w \tag{11}$$

The conservation of wall energy for each region is given in Equation 12 where $\dot{E}_w$ is the rate of change of the total energy of the heat exchanger wall in the region considered, $\dot{Q}_a$ is the rate of energy leaving the heat exchanger wall through heat transfer to the external fluid (generally air), $\dot{E}_{\text{int}}$ is the rate of energy being transferred to another region of the heat exchanger wall by a change in the boundary between the regions.

$$\dot{E}_w = \dot{Q}_w - \dot{Q}_a - \dot{E}_{\text{int}} \tag{12}$$

The conservation of mass for the entire heat exchanger is given in Equation 13 where the rate of change of the total refrigerant mass in the heat exchanger is equal to the difference between mass entering and exiting.

$$\dot{m} = \dot{m}_{in} - \dot{m}_{out} \tag{13}$$

For the gas cooler, these equations simplify to Equation 14, where $\dot{H}_c = \dot{U}_c + \dot{W}$, $\dot{W} = -A_{cs} L_{Total} \dot{P}_c$, $U_c = m_c u_c$, $E_w = (C_p \rho V)_w T_w$, and the right hand side is the same as $f(x,u)$, in Equation 6.

$$\begin{bmatrix} \dot{H}_c \\ \dot{m}_c \\ \dot{E}_w \end{bmatrix} = \begin{bmatrix} \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out} - \alpha_i A_i (T_r - T_w) \\ \dot{m}_{in} - \dot{m}_{out} \\ \alpha_i A_i (T_r - T_w) - \alpha_o A_o (T_w - T_a) \end{bmatrix} \tag{14}$$

This model is as simple conceptually as it is to derive. The state variables are defined as the total enthalpy of the refrigerant, the total refrigerant mass, and the total wall energy. This model can be shown to be equivalent to the model originally derived by expanding the variables $[\dot{H}_c \quad \dot{m}_c \quad \dot{E}_w]^T$, in terms of $[\dot{P} \quad \dot{h}_c \quad \dot{T}_w]^T$ and using the thermodynamic definition of enthalpy $h = u + P/\rho$. For details, see [9].

Another choice of state variables is $[P \quad m_c \quad T_w]^T$. The associated model is also found by expanding the variables $[\dot{U}_c \quad \dot{m}_c \quad \dot{E}_w]^T$. These two alternative models are given in the same form as Equation 5. The second model is defined by the states in Equation 15 and the matrix in Equation 16. The third model is defined by Equations 17 and 18. For all models $f(x,u) = f'(x,u) = f''(x,u)$. The advantages of each of these model representations will be discussed in Section 6. Although the process is slightly more complex due to the presence of two fluid regions, similar results can be derived for the evaporator. For details see [8].

$$x' = [P \quad m_c \quad T_w]^T \tag{15}$$

$$Z'(x,u) = \begin{bmatrix} \left(\left.\frac{\partial u_c}{\partial P_c}\right|_{\rho_c} \rho_c - 1 \right) & \left(u_c + \left.\frac{\partial u_c}{\partial \rho_c}\right|_{P_c} \rho_c \right) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (C_p \rho V)_w \end{bmatrix} \tag{16}$$

$$x'' = [U_c \quad m_c \quad E_w]^T \tag{17}$$

$$Z''(x,u) = I^{3x3} \tag{18}$$

4 ANALYSIS OF LINEARIZED MODEL

A classical approach is followed to obtain a linearized version of the model suitable for dynamic analysis and controller design. A Taylor series expansion is used where 2nd order and higher order terms are omitted and deviation variables are introduced. Space constraints will not allow a complete exposition of the linearized system, but complete details are available in [7]. This operating condition was selected to imitate highway-driving conditions while ensuring that the evaporator refrigerant outlet was superheated vapor. The operating condition is detailed in Table 1. SI units of [kg], [s], [m], [C], [kPa], and [kJ] are used for $\{A, B, C, D\}$.

Table 1: Highway Operating Condition

Compressor Speed	1800 [rpm]
Evaporator Air Flow Rate	300 [cfm]
Gas Cooler Air Flow Rate	2000 [cfm]
Indoor Ambient Temperature	32 [C]
Outdoor Ambient Temperature	50 [C]

Because of the equivalency of the three different model representations, the system matrices for the x' and x'' state representations are similarity transformations of the first (x). The resulting state matrices for each of the representations are given in Equations 19-21.

$$A_c = \begin{bmatrix} -16.202 & -1711.7 & 2706.5 \\ -0.31485 & -33.263 & 52.593 \\ 0.0030427 & 0.2912 & -0.6006 \end{bmatrix} \tag{19}$$

$$A_c' = \begin{bmatrix} -49.465 & 8.9885e6 & 2696.7 \\ 0 & 0 & 0 \\ 0.0087329 & -1534.9 & -0.6006 \end{bmatrix} \quad (20)$$

$$A_c'' = \begin{bmatrix} -49.465 & -6170.6 & 0.50826 \\ 0 & 0 & 0 \\ 46.335 & 5930.1 & -0.6006 \end{bmatrix} \quad (21)$$

The two alternative models A_c' and A_c'' expose the presence of a pure integrator associated with the accumulation of refrigerant mass. The eigenvalues confirm the presence of a zero eigenvalue, as well as indicating two-time scale behavior or a singularly perturbed system (Equation 22). A logical approximation would be a 2nd order model.

$$\lambda(A_c) = \begin{bmatrix} -49.943 \\ -0.12332 \\ 0 \end{bmatrix} \quad (22)$$

Evaluating the models for the evaporator also reveals possibilities for model reduction. The eigenvalues also indicate two-time scale behavior and an integrator (Equation 23). A logical approximation would be a 3rd order model.

$$\lambda(A_e) = \begin{bmatrix} -53.374 \\ -13.745 \\ -0.41128 \\ -0.13166 \\ 0 \end{bmatrix} \quad (23)$$

Similar analysis regarding the internal heat exchanger suggests that a 1st order model is sufficient. These possibilities are verified in Sections 6 and 7.

5 ANALYSIS OF EMPIRICAL MODEL

Previous research indicated that each individual input-output relationship could be adequately modeled with a 3rd order transfer function [8]. These models were developed using system identification techniques assuming a discrete-time ARMAX model. However, to verify the necessary dynamic order to adequately model the system dynamics MIMO dynamic models were developed using subspace system identification techniques. Models of order 1-10 were developed. The models of order 1-2 were inadequate for predicting the system response. The models of order 7-10 yielded results that adequately matched the data set used for estimation, but performed poorly when compared to a separate data set reserved for data validation, indicating that the model was over-fitted to the estimation data set. The 3rd and 4th order models adequately predicted the system response, but the correlation errors were unacceptable, indicating that the assumed noise was not white or independent of the control inputs. The 5th and 6th order models predicted the system response well, and correlation errors were the least achieved for any model order. From this, a 5th order model is concluded to be adequate for capturing the dominant system dynamics. Figure 3 shows the comparison between experimental validation data and the 5th order empirical model.

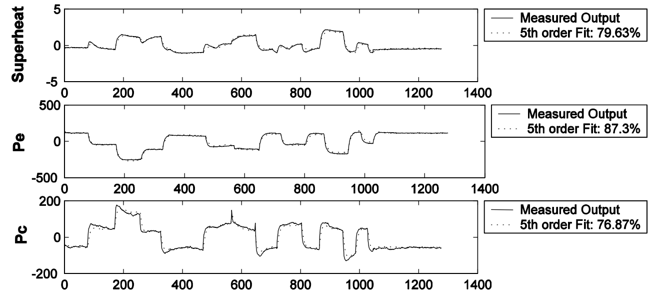

Figure 3: Evaporator Superheat, Evaporator Pressure, and Gas Cooler Pressure for Random Step Changes in All Inputs

6 MODEL REDUCTION

The analysis of the linearized models and empirical models motivates the investigation of reduced order models. The objective of this section is not only to derive reduced order component models, but also gain physical insight into which physical phenomenon are relatively fast/slow. To the authors' knowledge, the only known research on model-based order reduction of AC&R systems is in [4]. This reference, however, does not present simulation or validation results of the reduced order model, and no conclusions can be made regarding the validity of the simplifying assumptions.

Numerical methods of model reduction using balanced realizations transform the state variables and thus eliminate their physical meaning. On the other hand, singular perturbation techniques provide means for justifying the elimination of fast dynamic states without losing the physical nature of the states. The general form of a singularly perturbed system is given in Equation 24, where the perturbation parameter, ε, is assumed to be small, and x is assumed to represent the slow dynamics and z to represent the fast dynamics of the system.

$$\begin{aligned} \dot{x} &= f(x,z,\varepsilon,t) \\ \varepsilon\,\dot{z} &= g(x,z,\varepsilon,t) \end{aligned} \quad (24)$$

For some singularly perturbed systems the perturbation parameter, ε, may be known and appear explicitly in the equations. However, the models developed in this paper are singularly perturbed where ε is implicit and unknown. Additional difficulties include identifying which states are fast/slow and which of the available state representations is the best choice for model reduction. For the trivial case of the gas cooler model, reduced order models could be calculated for all possible representations and choices of fast/slow states, and the best choice could be selected by comparison. However, for more complicated models this method quickly becomes impractical. Therefore a method is presented here for: 1) selecting the "best" representation, 2) identifying the fast/slow states, and 3) calculating reduced order models. This procedure is applied to the gas cooler as an example.

The best representation is determined by analysis and observation. For the gas cooler, the first representation, x, resulting from the traditional modeling approach is a poor choice for model reduction because the pure integrator dynamic is not explicitly associated with any of the states. Both the second and third representations, x' and x'', contain the pure integrator explicitly. To compare these representations, the models are nondimensionalized (to ensure proper comparison) and the conditioning of the resulting A matrices is compared. The representation with the lowest condition number (ratio of largest to smallest singular value) is preferred. For the gas cooler the third representation is much less ill-conditioned than the second, with $cond(A') = 651524$ and $cond(A'') = 729.6$ (ignoring the zero singular value).

To identify the fast/slow states, the scaling matrix used to form a balanced realization is evaluated. Balancing the system matrices is performed by finding a scaling matrix S such that the induced norm of matrix M in Equation 25 is minimized. Relatively large entries in S indicate fast states. This is similar to visually inspecting the matrix and determining that the entries of a specific row are scaled by a large constant factor $1/\varepsilon$, and rearranging the equations into the standard form (Equation 24). For the gas cooler, the necessary scaling matrix for A_c'' is given as $S = diag([8 \quad 1 \quad 1])$. Thus the proper choice of the state to be residualized is the first state, or the refrigerant energy.

$$M = \begin{bmatrix} S^{-1}AS & S^{-1}B \\ CS & 0 \end{bmatrix} \tag{25}$$

After the fast/slow states are selected, a reduced order model can be calculated by residualizing the fast states. The state space form of the full order model is reordered into the form of Equation 26, and the reduced order model is calculated using Equation 27.

$$\begin{gathered}\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \\ y = [C_1 \quad C_2] \begin{bmatrix} x \\ z \end{bmatrix} + [D]u \end{gathered} \tag{26}$$

$$\begin{aligned} A_r &= A_{11} - A_{12} A_{22}^{-1} A_{21} \\ B_r &= B_1 - A_{12} A_{22}^{-1} B_2 \\ C_r &= C_1 - C_2 A_{22}^{-1} A_{21} \\ D_r &= D - C_2 A_{22}^{-1} B_2 \end{aligned} \tag{27}$$

To demonstrate that this choice of state representation does result in the best reduced order approximation, the eigenvalues of the reduced order models shown for each of the three state representations are compared to the dominant eigenvalues of the full-order system (Table 2). For the gas cooler there is only one dominant eigenvalue, and both the second or third state representations approximate it equally well. For the evaporator there are two dominant eigenvalues, and the third state representation results in the best approximation.

Table 2: Reduced Order Approximation of Dominant Eigenvalues

	Gas Cooler			
	Full Order Model	Reduced Order Models		
		X	X'	X''
Dominant Eigenvalue	-0.123	-0.140	-0.125	-0.125

	Evaporator			
	Full Order Model	Reduced Order Models		
		X	X'	X''
1st Dominant Eigenvalue	-0.411	-0.914	-0.375	-0.427
2nd Dominant Eigenvalue	-0.132	-0.062	-0.151	-0.141

Reduced order models are calculated for the gas cooler, evaporator, and internal heat exchanger. After combining these reduced order component models, the resulting system model approximates well the eigenvalues of the full-order system model. Furthermore, one additional state can be removed because of a modeling redundancy. There is a conservation of mass equation for both the gas cooler and evaporator. However, because the net loss in refrigerant mass for one of these components equals the net mass gain of the other, these two dynamic equations are redundant and one can be removed. Thus the original 11th order model is reduced to a 5th order model. More importantly, residualization of the alternative models yielded the insight that the dynamic modes associated with the refrigerant energy states (pressure, moving boundary length, etc.) are fast compared to the wall temperature dynamics and the location of refrigerant mass, and can be replaced with algebraic equations.

7 MODEL VALIDATION

To validate the derived models, a test facility [6] is used to record transient system responses based on changes in control inputs. A pseudo-random binary sequence is applied to each of the four controllable inputs individually and simultaneously. The transient data is compared to simulated results for all models in Figures 4-6. The data is shown for the operating condition defined in Table 1.

The comparison between simulation results and experimental data shows partial agreement. The predicted evaporator exit air temperature shows a discrepancy in steady state value (Figure 4). The dynamic response of evaporator superheat matches in form but not in magnitude (Figure 5). Evaporator pressure shows good agreement (Figure 6). For all outputs, the 11th order linearized model approximates closely the nonlinear model, with the small discrepancies caused by linearization assumptions used in the internal heat exchanger. The 5th order model is indistinguishable from the full-order linearized model, indicating that residualizing six dynamic modes did not compromise the model fidelity.

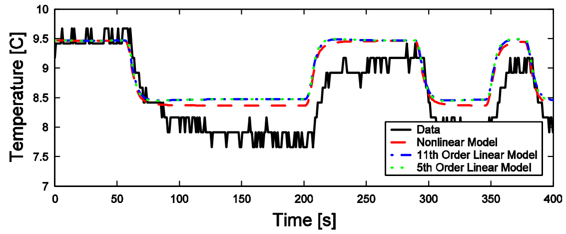

Figure 4: Evaporator Exit Air Temperature for Step Changes in Compressor Speed

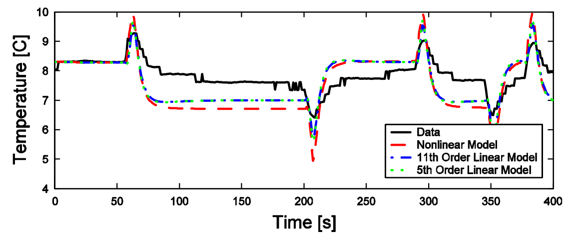

Figure 5: Evaporator Superheat Temperature for Step Changes in Compressor Speed

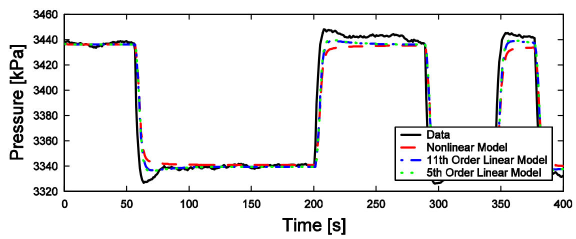

Figure 6: Evaporator Pressure for Step Changes in Compressor Speed

8 CONCLUSIONS

This paper makes several key contributions to the study of vapor compression system dynamics. First, an existing modeling approach is applied to a unique type of vapor compression cycle. Second, a different modeling approach is presented and shown to be equivalent to the more common approach, but simpler conceptually and to derive. This alternative approach also exposed freedom in choosing the system states. Third, analysis of the linearized models and the empirical models constructed using system identification techniques indicates that a reduced order model of the system dynamics is adequate. Finally, variations of the singular perturbation technique were used to find physically based reduced order component models. To the authors' knowledge, this is the first model-based explanation of order reduction with demonstrated validation. The more commonly derived models were shown to be inappropriate for model reduction, while the reduced order models using the alternatively derived models were shown to result in good approximations of the full order system, as well as expose a redundant dynamic mode. All models were validated against experimental data taken at a testing facility available at the University of Illinois at Urbana-Champaign.

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APPENDIX

Variable	Explanation	Subscript	Explanation
T	Temperature	f	Liquid
P	Pressure	g	Vapor
h	Enthalpy	i	Inner
u	Internal Energy	o	Outer
C_p	Specific Heat	T	Total
ρ	Density	ave	Average
α	Heat Transfer Coefficient	cs	Cross-Sectional
$\dot{m}$	Mass Flow Rate	r	Refrigerant
A	Area	w	Wall
L	Length	e	Evaporator
V	Volume	c	Gas Cooler