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APPLICATION OF MULTIVARIABLE ADAPTIVE CONTROL TO AUTOMOTIVE AIR CONDITIONING SYSTEMS

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ABSTRACT

This paper presents the application of a multivariable adaptive control strategy to a typical automotive air conditioning system. First, an experimentally validated physical model for the air conditioning cycle is introduced. This is followed by the application of a multi-input multi-output (MIMO) parameter estimation algorithm to recursively identify an equivalent discrete time state space model of the system. A Linear Quadratic Regulator (LQR) design is implemented on the estimated model with the objectives of reference tracking and disturbance rejection. Simulation studies are performed to explore the idea of modulating the electronic expansion valve opening and air flow rate over the evaporator for controlling the efficiency and capacity of a general automotive air conditioning unit. The results demonstrate the efficacy of the MIMO controller for these objectives.

1. NOMENCLATURE

Variable	Definition	Variable	Definition
A,B,C	State Space Matrices	h	Enthalpy
J	Cost Function	k	Discrete Time Step
K	State Feedback gain	p	Pressure
L	Kalman Gain Matrix	t	Time
P	Covariance Matrix	u	Input
Q , R	Weighing Matrices	x	States
T	Temperature	y	Output
U	Fluid Velocity	$\hat{y}$	Estimated Output
V	Volume	z	Length Dimension
a	Area	α	Heat Transfer Coeff.
c_v	Orifice Factor	ε	Innovations Vector
d	Pipe Diameter	$\hat{\varepsilon}$	Aposteriori Error
e	Apriori Error	ρ	Density

Variable	Definition	Subscript	Definition
Δ	Gradient	j	jth Output
ω	Compressor Speed	k	Compressor
θ	Parameter Vector	kri	Compressor Inlet
Subscript	**Definition**	kro	Compressor Outlet
1	Region 1	n	Observability Index
2	Region 2	o	Outlet
ai	Air	r	Refrigerant
c	Condenser	ss	Steady State
e	Evaporator	v	Valve
i	Inlet	w	Wall

2. INTRODUCTION

A majority of air conditioning and refrigeration devices operate using a vapor compression cycle. A typical subcritical cycle is shown in Fig. 1.

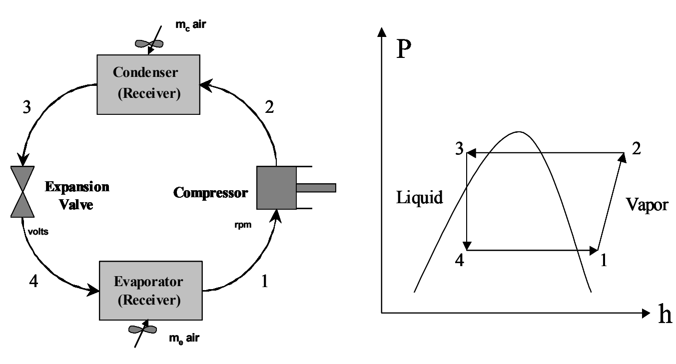

Figure 1: Subcritical Vapor Compression Cycle

The working fluid absorbs heat as it evaporates, and then is compressed to a high pressure where heat is rejected as the fluid condenses [1]. The difficulty of modeling the complex thermofluid dynamics associated with these phase changes has

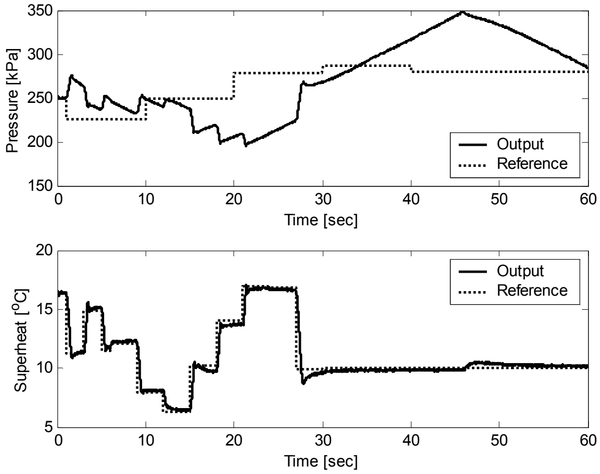

Figure 2: Dual SISO Loop PI Control of the A/C System

restricted the development of advanced controllers for these systems [2]. Recent advances in this field of research have resulted in increase of control authority with the development of variable displacement compressors, electronic expansion valves, and variable speed fans for air flow over the heat exchangers. A well designed control strategy is important for effective use of all these actuators. Further, the design of control systems for automotive air conditioning is primarily driven by concern for unit efficiency and capacity control despite the transient disturbances acting on the system. Refrigerant superheat (a measure of excess temperature of the refrigerant above saturation) at the evaporator outlet [1] is used as a relative measure of efficiency. The objective is to maximize the area in the heat exchanger with liquid refrigerant contact while ensuring that liquid does not enter the compressor. Similarly, refrigerant inlet temperature (or pressure) to the evaporator is used as a relative measure of heat transfer capacity. Multiple single-input single-output (SISO) control loops can be designed to achieve these performance objectives with a proper choice of actuators. However, as described in [2,3], strong cross-coupling exists between different input-output pairs of a vapor compression cycle and multiple SISO control loops cannot compensate for such coupled dynamics. This is demonstrated in Fig. 2, where a simulation study on reference tracking of pressure and superheat using dual SISO controller for an air conditioning system model is presented.

While details of the system model structure will be given in Section 3, Fig. 2 directly illustrates that due to coupling of the system's dynamics accurate control of superheat interferes with the pressure loop and causes sub-standard control of pressure. As pointed out by [2,3] this cross-coupling can be incorporated in the design of a multivariable controller for better performance. A model-based MIMO controller design for vapor compression systems is presented in [2]. Experimental work on controllers for subcritical systems was performed by Dane [4], where a drastic reduction in power consumption was obtained by using a variable speed compressor as a control actuator along with an electronic expansion device. Similarly, modulating the speed of the variable speed airflow fans over the heat exchangers can also play an important role in capacity control in transient conditions [2]. Although not typically used in production vehicles, this technology is currently being explored by various automotive industries [5]. Forrest and Bhatti [5] have presented an analogous approach towards energy efficiency by appropriately mixing the re-circulated and incoming air to the evaporator in an automotive system. Building on this research, the current paper explores the performance of coordinated MIMO control of the expansion valve and the air flow rate over the evaporator for achieving the performance objectives.

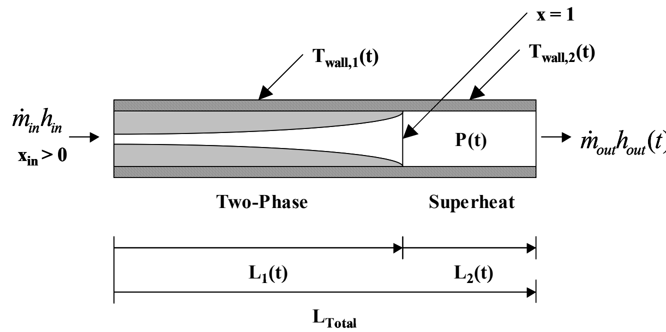

Figure 3: Different Regions of Heat Transfer

Past efforts using adaptive control techniques for air conditioning systems have included numerous self-tuning PID algorithms for SISO control with the output generally being air temperature [6]. Some more advanced multivariable adaptive control algorithms have also been successfully applied in past. The application of these algorithms is generally for controlling the temperature of different "zones" in a building for intelligent climate control [7]. Similarly, coordinated self-tuning control of air temperature and humidity is presented in [8]. The unique contribution of this paper is the application of a multivariable adaptive control technique on the primary refrigerant circuit, rather than the air loop, for efficiency and capacity control of an automotive air conditioning unit. Simulation studies for controller performance using the expansion valve opening and the evaporator air flow rate as actuators are also presented and the results demonstrate that a multivariable controller can perform better than multiple loops of SISO controllers.

The remainder of this paper is organized as follows. Section 3 details the physical model description and presents validation results, which justify some of the numerical conclusions to be presented in subsequent sections. This is followed by a description of the multivariable state space adaptive identification scheme and controller design based on an LQR methodology in Section 4. Results and discussion of reference tracking and disturbance rejection scenarios are provided in Section 5. Conclusions and plans for future research form the last section of this paper.

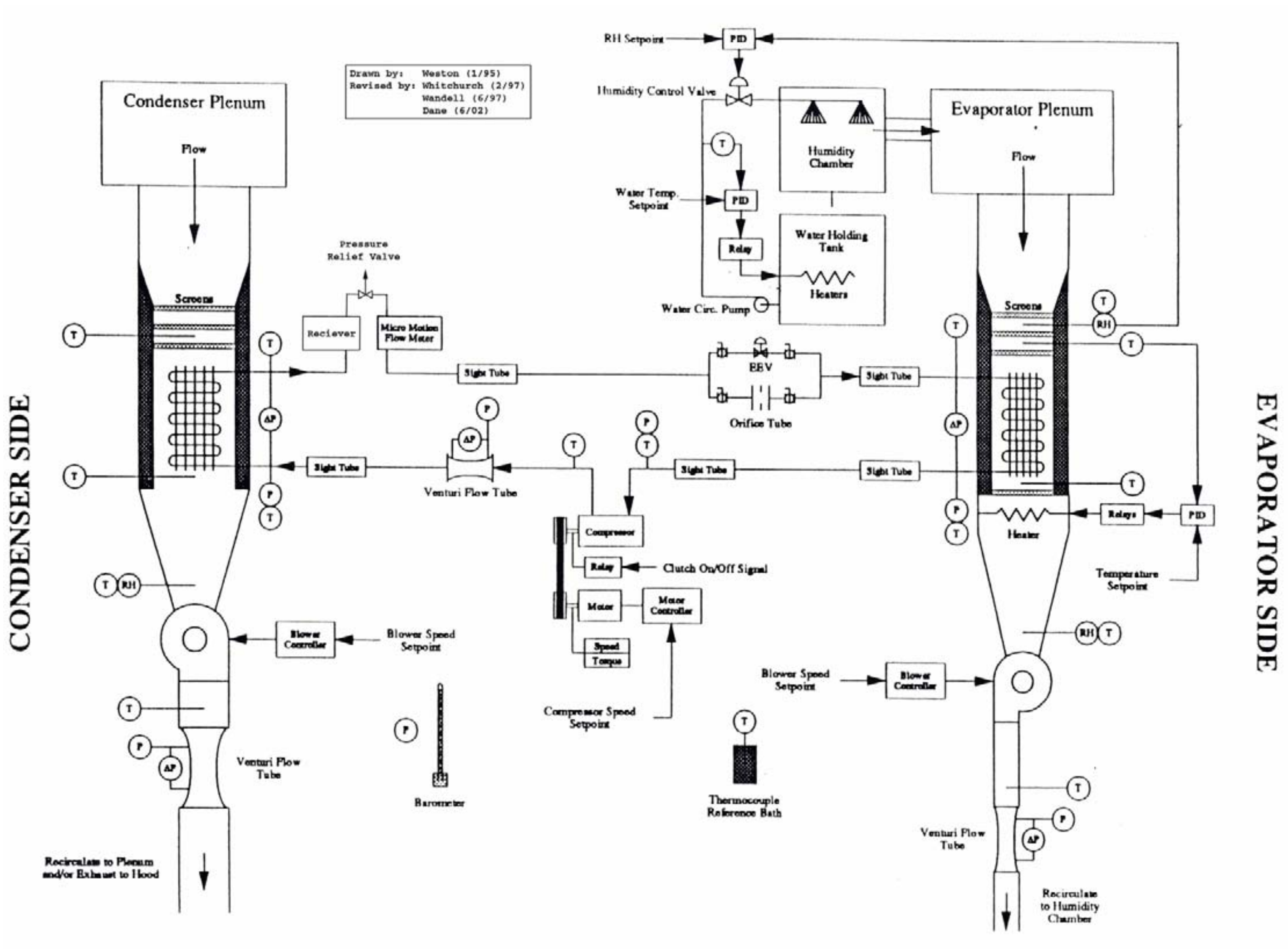

Figure 4: Experimental System for Automotive A/C at UIUC ([4])

3. PHYSICAL MODELING

The physical model of a subcritical vapor compression cycle is based on lumped parameter methods for heat exchanger modeling [2,3]. In this approach, the heat exchanger is allowed to have time varying lengths of different regions of sub-cooled, two-phase, and superheated flow. Figure 3 shows the refrigerant flow in an evaporator consisting of two regions. The refrigerant enters as a two phase liquid and leaves as a superheated vapor. Similarly, the flow phenomenon in a condenser has three regions of superheated, two-phase, and subcooled flow [9]. Generalized Navier-Stokes equations [10] can be used to express the refrigerant properties in these different regions. An important assumption is that the friction losses in the heat exchangers are negligible which renders the momentum conservation equation redundant [2]. The equations for the refrigerant mass and energy balance and the wall energy balance can then be written as,

Mass Balance:
$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho U)}{\partial z}=0 \qquad (1)$$

Energy Balance:
$$\frac{\partial(\rho h-p)}{\partial t}+\frac{\partial(\rho U h)}{\partial z}=\frac{4}{d_i}L_i\alpha_i\left(T_{wall}-T_r\right) \qquad (2)$$

Wall Energy Balance:
$$\left(C_p\rho V\right)_w \dot{T}_w=\alpha_i a_i\left(T_r-T_{wall}\right)+\alpha_o a_o\left(T_w-T_{wall}\right) \qquad (3)$$

The resulting dynamic model for an evaporator is of 5th order with the dynamic modes being length of two phase flow L_1, refrigerant pressure p_e, refrigerant outlet enthalpy h, and wall temperatures in the saturated and superheated region, T_{w1} and T_{w2}, respectively [3].

$$x_e=\begin{bmatrix}L_1 & p_e & h & T_{w1} & T_{w2}\end{bmatrix}^T \qquad (4)$$

Similarly, the condenser is a 7th order system with dynamic modes being length of the two condensation regions L_1 and L_2, refrigerant pressure p_c, refrigerant outlet enthalpy h, and the wall temperatures in the three regions, T_{w1}, T_{w2} and T_{w3} respectively [9].

$$x_c=\begin{bmatrix}L_1 & L_2 & p_c & h & T_{w1} & T_{w2} & T_{w3}\end{bmatrix}^T \qquad (5)$$

The time constant for the dominant dynamics of the expansion valve and the compressor are much shorter than those of the heat exchangers [2,3]. Thus, via time scale separation reasoning, these components can be modeled using static algebraic equations. The expansion valve can be modeled by the orifice equation, with c_v being the coefficient of discharge.

$$\dot{m}_v = c_v a_V \sqrt{\rho_v \Delta p} \tag{6}$$

The compressor mass flow rate is modeled as,

$$\dot{m}_k = \omega_k V_k \rho_k \left(1 + A_k - B_k \left(\frac{p_{kro}}{p_{kri}} \right)^{1/r} \right) \tag{7}$$

Here, A_k and B_k are volumetric efficiency coefficients for the compressor and r is the polytropic compression coefficient for the refrigerant. These individual component models are integrated to design the complete system model which is 12th order (five dynamic modes from the evaporator model and seven from the condenser) for a subcritical automotive air conditioning cycle. The model of the system was created and simulated via Thermosys^{TM} [11], a Matlab/Simulink based environment for simulating Vapor Compression Cycle systems. Thermosys^{TM} is a dynamic modeling tool that has been created by the authors through the support of the Air Conditioning and Refrigeration Center (ACRC) at the University of Illinois, Urbana-Champaign [11]. The model was validated against a subcritical automotive air conditioning test system that is also part of the ACRC. The experimental system has a variable speed motor-driven compressor, an electronic expansion valve, and variable frequency driven air fans for heat exchangers. Figure 4 shows some of the flow straightening devices along with an array of pressure sensors as part of the transducer suite available for data acquisition. For a sense of scale, the entire system of Fig. 4, including the accessories to condition the air temperature/humidity/etc., occupies a space approximately 5 meters by 5 meters. A complete description of the system parameters is available in [9].

The transient behavior of the physical model described above was validated against the data from this set-up for different experiments. One such validation set is shown in the Figures 5 and 6 for step changes in expansion valve opening. These validation graphs show that the simulation model is able to reflect the transient response of the air conditioning system very well. While the steady-state values of the simulation may be slightly off from the data, this is acceptable in order to have a model capable of capturing transient dynamics. Other commercially available simulation tools, which are capable of generating better steady-state matching, are not able to provide the transient modeling performance. The relative dynamic accuracy of the simulation is essential to the adaptive MIMO results to be presented later.

The experimental facilities at the ACRC are well calibrated for accuracy in system characterization and testing, thus ensuring that the experimental results used for model validation are comparable to a real system [4]. Therefore further study of the controller implementation and performance evaluation for reference tracking and disturbance rejection tests were performed within the Thermosys^{TM} environment. Previous dynamic analysis of the model reveals that this system is singularly perturbed [3,12]. This means that the dynamics of the thermo-fluid system evolve on different time-scales with some fast dynamics modes and some slow ones. Much like the valve and compressor equations, these fast dynamics can be replaced by algebraic relationships. In [3], a quantitative evaluation of dominant dynamics of a vapor compression system is presented where it is shown that the wall temperatures and the mass accumulation of the refrigerant are the most dominant dynamics. Dynamics such as the transfer or storage of energy within the refrigerant fluid itself were found to be much faster than the mass accumulation and heat exchanger thermal capacitance dynamics. This was verified both numerically and experimentally [3,9].

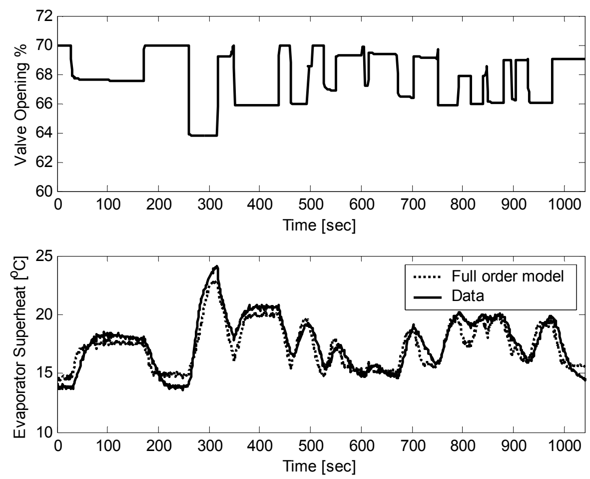

Figure 5: Time Domain Model Validation

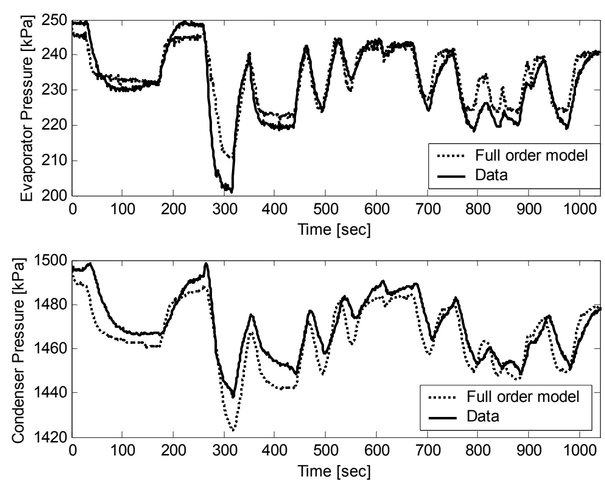

Figure 6: Time Domain Model Validation

Using the procedure outlined in [3,12] a reduced order model of the system was found by residualizing the fast dynamic modes of the individual component models. This results in a 6^{th} order model that is completely controllable and observable as shown in [9]. This 6^{th} order model is also effective for predicting the transient response of all associated system variables due to the actuators of interest. However, in this study, only two inputs and two outputs will be used for controller design. Based on engineering insight to the dynamics of this system, only four of the dynamic modes strongly participate in the dynamics of the selected inputs/outputs. Therefore, a 4^{th} order model structure is selected for identification, which reduces the number of parameters to be identified significantly.

4. ADAPTIVE ESTIMATION AND CONTROL

4.1 Motivations for Adaptive Control

The ability to accurately simulate a dynamic system facilitates the design of a model-based controller. For automotive air conditioning systems, the physical model is highly nonlinear because of fluid properties and thermodynamic relations and hence model linearization is required for useful model-based controller design. This has been successfully shown in [2] where LQR controller design on a linearized model is presented. However, the linearized model will only be valid in some range about a given operating condition. Possible ways to compensate for the model nonlinearities include:

- Robust control design for the linearized system [13]
- A gain scheduled controller for the linearized system.
- Adaptive identification and control of the nonlinear model.

Of these methods, the first two require physical knowledge of the system over a relatively wide operating range. This is necessary to estimate the uncertainty bounds for the robust controller or to know the plant parameters at several operating points and schedule controllers for the entire operating range [2]. Alternatively, an adaptive control approach can be used with different systems over varying sets of parameters and varying operating conditions. With each parameter or operating condition change, the adaptation algorithm estimates the corresponding new linearized model to be used for controller design. This is quite practical for an automotive air conditioning system which has to go through disturbances like highway cruising, or idling, or city stop-and-go traffic. Many of the external conditions such as temperature, humidity change slowly as compared to the dynamics of an air conditioning unit. Moreover, the time required for adaptation convergence to a condition change will be much smaller than the time an automotive vehicle will be driven in the new set of conditions. Following this reasoning it can be said that a well-designed adaptive controller can compensate for the system nonlinearities.

Another motivation for adaptive control lies in the fact that, in certain cases, true parameter values may not be available. For example, clogging in heat exchanger pipes, lower efficiency of the compressor, and other faults can result in a need to retune a model-based controller. However, an adaptive controller will estimate the appropriate system parameters, given sufficiently rich excitation signals, and a corresponding controller can be designed. Even if the parameters don't converge to their true value, the predictive capability of the algorithm will be sufficient for the controller design.

4.2 Estimation Algorithm

As described in Section 3, a 4^{th} order dynamic model was chosen based on the dynamic analysis of the physical model of the system. Of various candidate model structures, a state-space model structure was chosen for adaptive identification because of the advantage of easy implementation of various control strategies through state feedback. Indirect adaptive control design is then a three step problem comprising of parameter identification, state estimation, and controller implementation. A modified recursive bootstrap algorithm (RBA) [14] and a recursive prediction error method (RPEM) [15] are two widely used methods for online identification of state space models. In both these methods, simultaneous state and parameter estimation is performed. However, it is argued that because of the nonlinearity of RPEM, true parameter convergence is not achieved [14]. The RBA splits the nonlinear problem into two linear ones using the certainty equivalence principle and consistency is achieved with a high probability as proved in [14]. In the current work, we present system identification using the modified RBA by coupling it to an extended least squares algorithm.

For recursive identification, the air conditioning system model can be represented by a stochastically disturbed discrete time state space equivalent [14] as,

$$\begin{aligned} \hat{x}(k+1,\theta) &= A(\theta)\hat{x}(k,\theta) + B(\theta)u(k) + L(\theta)\varepsilon(k) \\ \hat{y}(k) &= C(\theta)\hat{x}(k,\theta) + \varepsilon(k) \end{aligned} \tag{8}$$

Here, $\hat{x}$ is the n-dimensional state vector, u is the r-dimensional input vector, $\hat{y}$ is the m-dimensional output vector and ε is the m-dimensional vector of innovations. The elements of system matrices $A(\theta)$, $B(\theta)$, $C(\theta)$ and the Kalman gain matrix $L(\theta)$ are the set of parameters desired to be identified. Simultaneous estimation of the Kalman gain matrix avoids the solution of the standard state estimation algorithm [14], which makes the identification simpler.

In the present study, the problem of controlling the superheat and refrigerant pressure at the evaporator inlet by modulations in the expansion valve opening and the evaporator air fan speed results in a dual input-dual output system. Closed loop identification is obtained by providing a persistently exciting signal as the output reference. A white noise sequence is also added as measurement noise at the outputs. The estimation algorithm is applied to the observability canonical representation [14] in which A and C can be expressed as given below.

$$A = \lfloor A_{ij} \rfloor_{nxn} \quad i, j = 1, \ldots m$$

$$A_{ii} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ . & . & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{ii,1} & a_{ii,2} & . & a_{ii,n_i} \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & . & . & 0 \\ 0 & . & . & 0 \\ a_{ij,1} & a_{ij,2} & . & a_{ij,n_{ij}} \end{bmatrix} \tag{9}$$

$$C = \begin{bmatrix} e_1 \\ e_{n_1+1} \\ \ldots \\ e_{n_1+n_2+\ldots\ldots+n_{j-1}+1} \end{bmatrix} \tag{10}$$

Here, e_n is a unit row vector with the nth element as 1. The jth output of the model can then be written as,

$$\hat{y}_j(k+1) = \theta_j^T(k)\Phi_j(k) + \varepsilon_j(k+1) \tag{11}$$

$\Phi_j(k)$ is the regression vector comprised of estimated states, inputs and innovations given as,

$$\Phi_j(k) = \begin{bmatrix} \hat{x}^T(k-n_j+1)\ldots\ldots u^T(k-n_j+1)\ldots\ldots u^T(k) \\ \hat{\varepsilon}^T(k-n_j+1)\ldots\ldots \hat{\varepsilon}^T(k) \end{bmatrix}^T \tag{12}$$

The regression vector is generated by recursively estimating the states $\hat{x}$ obtained from the stochastic process description shown in Eq. 8 above. The innovations $\hat{\varepsilon}$ are also estimated by aposteriori error calculation using an extended least squares algorithm. The parameter vector $\theta_j(k)$ has the elements of $A(\theta)$, $B(\theta)$ and $L(\theta)$ as,

$$\theta_j(k) = \begin{bmatrix} a_{j1,1}\ldots\ldots a_{j1,n_1} & a_{j2,1}\ldots\ldots a_{j2,n_2} & \ldots\ldots & a_{jm,1}\ldots\ldots a_{jm,n_m} \\ b_{n_1+\ldots+n_j,1}\ldots\ldots b_{n_1+\ldots+n_j,r} & \ldots\ldots & b_{n_1+\ldots+n_{j-1},1}\ldots\ldots & b_{n_1+\ldots+n_{j-1},r} \\ l_{n_1+\ldots+n_j,1}\ldots\ldots l_{n_1+\ldots+n_j,r} & \ldots\ldots & l_{n_1+\ldots+n_{j-1},1}\ldots\ldots & l_{n_1+\ldots+n_{j-1},r} \end{bmatrix}^T \tag{13}$$

where $b_{p,q}$ and $l_{p,q}$ are the (p,q) elements of $B(\theta)$ and $L(\theta)$ respectively. With this structure, simultaneous parameter and state estimations is done by using the extended least squares algorithm [14,16] and [17]. In this algorithm, the apriori error is first obtained by

$$e(k) = \left(\hat{y}_j(k) - \hat{\theta}_j^T(k-1)\Phi_j(k-1)\right)\beta^{1/k} \tag{14}$$

Here β is a small positive number that acts as a learning factor [14] to avoid parameter divergence during the initial estimation when errors are big. The exponent k is the time since the beginning of adaptation and hence the multiplying factor approaches one as time increases. The apriori error is then used for updating the parameters as,

$$\hat{\theta}_j(k) = \hat{\theta}_j(k-1) + \alpha_j P_j(k-1)\Phi_j^T(k-1)e(k) \tag{15}$$

Here, P is the covariance matrix of the system. Since a linearized model for the air conditioning system is a time-varying system, a covariance resetting technique [16,17] is applied for better system estimation. In this approach, the resetting variable α_j is varied on the basis of richness of the innovations vector. The resetting is carried out as follows:

$$\alpha_j = \begin{cases} 0, \text{if } e_k = 0, (\text{reset } P_j) \\ 1, \text{otherwise} \end{cases} \tag{16}$$

This is followed by state estimation using Eq. 8, the aposteriori error calculation using Eq. 11, generation of a new regression vector using Eq. 12, and covariance matrix updating as shown in Eq. 17 below,

$$P_j(k) = P_j(k-1) - \frac{P_j(k-1)\Phi_j(k)\Phi_j^T(k)P_j(k-1)}{1 + \Phi_j^T(k)P_j(k-1)\Phi_j(k)} \tag{17}$$

Proof of true parameter convergence of this estimation algorithm is presented in [14] under typical assumptions such as persistence of excitation. This algorithm is recursively applied for identification of the 4th order state space model of the system under consideration. The number of observability indices associated with superheat and pressure were considered to be three and one respectively. The total number of parameters for a 4th order model is 24 (eight each for *A(θ), B(θ)* and *L(θ)* respectively). For a physical system, the number of individual parameters is larger than 24 [2,9], but 'grouping' of these physical parameters occurs because of various thermodynamic and physical relations. The results of the identification procedure can be seen in Figures 8 and 9 in Section 5. These results show the input-output behavior and parameter convergence of this scheme for the identified fourth order model.

4.3 Controller Design

An optimal LQR tracker design [18,19] is implemented with the following cost function based on output weighing for the identified MIMO plant.

$$J = \frac{1}{2}\sum_{k=0}^{N-1}\left[y^T(k)Qy(k) + u^T(k)Ru(k)\right] \tag{18}$$

The weighing matrices Q, and R are constant real and symmetric. Q is assumed to be positive semi-definite and R as positive definite. The elements y are the outputs of the model. In order to generalize the controller for discrete time reference tracking, an error state augmentation scheme is proposed. It is possible to define an integral error state as the difference between the outputs and the references as,

$$\begin{aligned} x_i(k+1) &= x_i(k) + y(k+1) - r(k+1) \\ &= x_i(k) + C(k)x(k+1) - r(k+1) \end{aligned} \tag{19}$$

The integral error states can be augmented to the original state vector and the resulting dynamic system can be represented as,

$$\begin{bmatrix} x(k+1) \\ x_i(k+1) \end{bmatrix} = \begin{bmatrix} A(k)x(k) + B(k)u(k) + L(k)\varepsilon(k) \\ x_i(k) + C(k)A(k)x(k) + C(k)B(k)u(k) + C(k)L(k)\varepsilon(k) \\ + \varepsilon(k+1) - r(k+1) \end{bmatrix} \tag{20}$$

Or,

$$\begin{bmatrix} x(k+1) \\ x_i(k+1) \end{bmatrix} = \begin{bmatrix} A(k) & 0 \\ C(k)A(k) & I \end{bmatrix}\begin{bmatrix} x(k) \\ x_i(k) \end{bmatrix} + \begin{bmatrix} B(k) \\ C(k)B(k) \end{bmatrix}u(k) + \begin{bmatrix} L(k) \\ C(k)L(k) \end{bmatrix}\varepsilon(k) + \begin{bmatrix} 0 \\ I \end{bmatrix}\left(\varepsilon(k+1) - r(k+1)\right) \tag{21}$$

Since the states are generated by the identification and are not known physically, the output weighing based controller was obtained using the cost function in Eq. 18. The controller gains can be obtained using the discrete time subroutine available in Matlab® and invoked using the command dlqry. In order to use the error states as the outputs for the controller design, we define a new output-state matrix such that,

$$\tilde{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

Now, we can use the A and B matrices from Eq. 21 along with this matrix and a zero direct feed through matrix for obtaining the control effort through the LQR design. Once the output error converged to zero, the output and input gains given by Eq. 23 were used for the current study.

$$Q = \begin{bmatrix} 1e-2 & 0 \\ 0 & 1.25e-2 \end{bmatrix}, R = \begin{bmatrix} 3.5e+5 & 0 \\ 0 & 2e+6 \end{bmatrix} \tag{23}$$

Here the orders of the outputs and inputs are pressure, superheat, and valve opening, air flow rate respectively. The augmented states are obtained as,

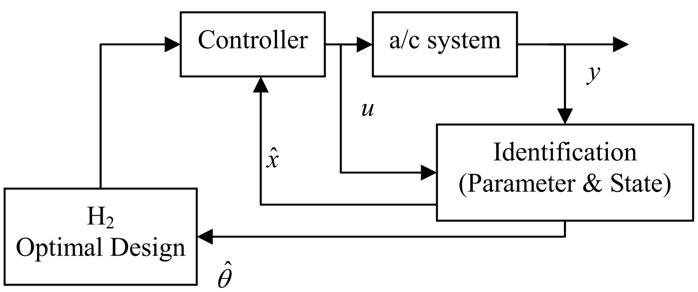

Figure 7: Block Diagram of the Implemented Adaptive Control

$$x_{aug} = \begin{bmatrix} x(k) \\ x_i(k) + y(k) - r(k) \end{bmatrix} \tag{24}$$

It should be noted here that the error states are obtained using the true plant outputs y rather than the estimated output $\hat{y}$ because the goal is to bring the true system close to the references rather than the identified model. The final control effort can then be obtained using the gains K calculated by discrete LQR design and the augmented states as,

$$u(k) = -K(k)x_{aug}(k) \tag{25}$$

5. RESULTS AND DISCUSSION

The estimation algorithm and controller design for the physical model was implemented in MATLAB/Simulink® and combined with the Thermosys™ simulation environment as shown in Fig. 7. Input-output convergence and parameter convergence for closed loop identification is depicted in Figs. 8 and 9. For closed loop identification, reference signal should be persistently exciting of sufficient order [16,17]. In this study, a random noise input with a variance of 40 kPa about a mean of 280 kPa was applied as a reference signal for the evaporator pressure. Similarly for superheat reference, a random noise signal of variance 0.01 degrees Celsius was applied about a mean of 16 degrees Celsius. This ensures persistence of excitation for the closed loop system. Further, white noise sequences with variances of 0.005 and 0.0015, respectively, were added at the two output measurement channels for pressure and superheat. In Fig. 9, we see that the parameters have converged to steady state values for this particular set of conditions. Also, the input-output transfer function match is visible from the graph in Fig. 8. The estimated outputs begin from zero and converge slowly in the beginning because of the exponential learning factor in Eq. 14. The next step was to implement the LQR tracker presented in Section 4.3 for the identified model in Fig. 9 to evaluate its reference tracking and disturbance rejection characteristics. Figures 10 depict the performance of the MIMO adaptively controlled air conditioning cycle for reference tracking. A sequence of step changes in the reference signal is applied for the evaporator pressure and the controller demonstrates good performance throughout by bringing the true outputs to their desired values.

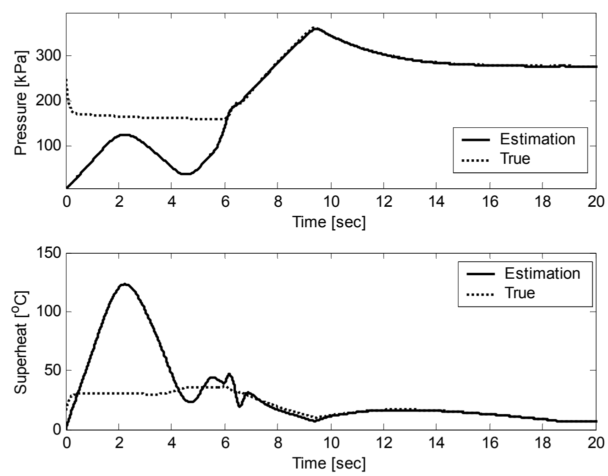

Figure 8: Input-Output Convergence Results

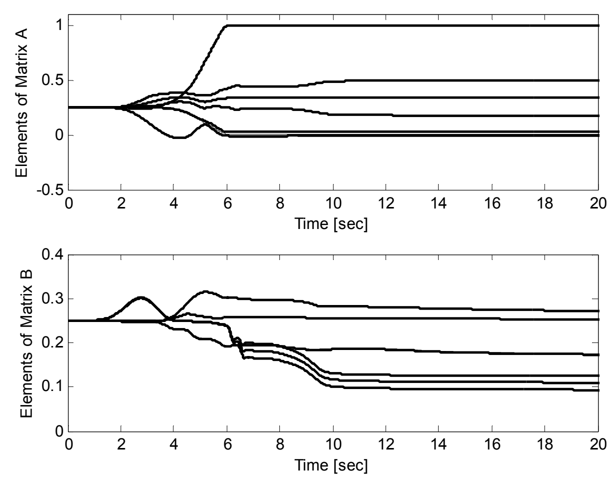

Figure 9: Parameter Convergence

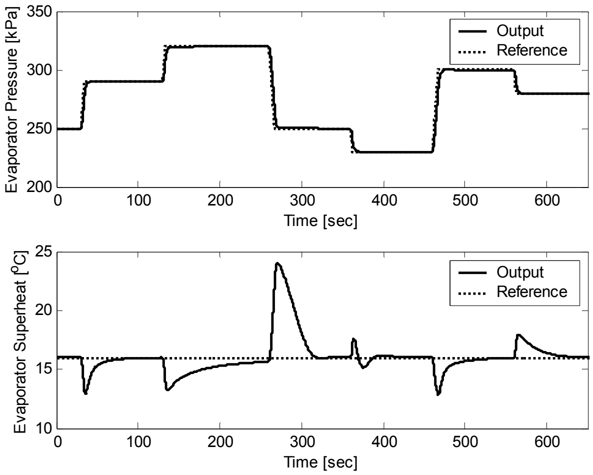

Figure 10: Reference Tracking of Evaporator Pressure

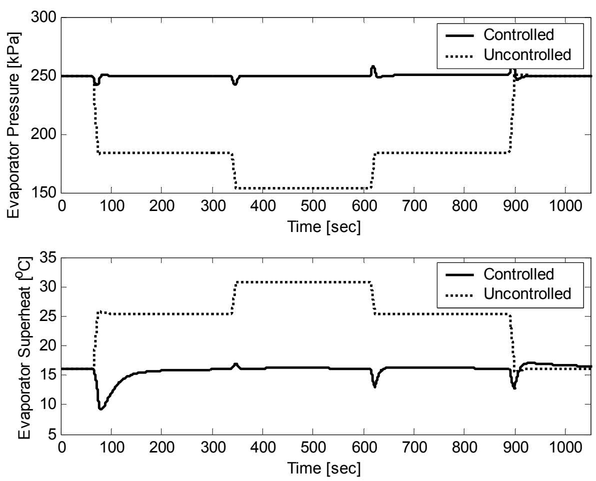

Figure 11: Disturbance Rejection Characteristics

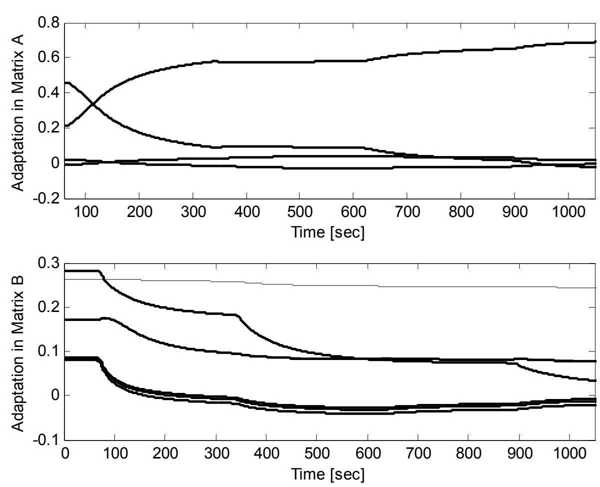

Figure 12: Parameter Variation due to Changes in Operating Conditions

For evaluating the disturbance rejection characteristics of the controller, changes in driving condition are applied to the system, as studied in [4]. Rotational speed of the compressor and condenser side air flow rate are changed to emulate the change in driving condition from idling to city-driving to highway driving and back. The same set of disturbances is applied to an uncontrolled model to evaluate the absolute performance of the controlled plant. The comparison is shown in Fig. 11. We can see that the MIMO adaptively controlled plant is able to model the strong coupling and provide the appropriate control action as expected. The motivation of using an adaptive control approach also becomes clear in this study. Because of the changes in the operating conditions due to the disturbances, the identification algorithm adapts to different values of the parameters as shown in the Fig. 12. This justifies the use of an adaptive approach for an automotive air conditioning problem. Furthermore, from Fig. 11, it can be observed that the disturbance rejection properties of the controller improve as the adaptation progresses.

6. CONCLUSIONS AND FUTURE WORK

The principle contributions of this paper are as follows. First, it presents application of a multivariable adaptive control strategy to a general automotive air conditioning system. Secondly, it explores the possibility of combined modulations of both the expansion valve opening and airflow rate over the evaporator as a control strategy for vapor compression systems. It was demonstrated that the MIMO adaptive approach was very successful at achieving multiple objectives by regulating both superheat and pressure in the evaporator. All identification and control results were demonstrated in a detailed numerical simulation environment that had been verified by data taken from a production automotive air conditioning system. Future research will include implementation of these control strategies on an experimental test-bed with varying system parameters to evaluate the adaptive capabilities of the system. Another area of future research would be performance evaluation and viability of this control strategy against other industrial controllers in terms of performance and power consumption. Although outside the direct scope of this paper, the current adaptive control scheme can be developed for fault detection also. The system could be integrated with a logic database for diagnosis of various faults in the system and changes in identified parameters can be monitored for detection of these faults.

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REFERENCES

[1] Jordan, R. C., 1956, *Refrigeration and Air Conditioning*, Prentice Hall Inc., Englewood Cliffs, New Jersey, pp. 16-31,161-186,345-367.

[2] He, X.-D., Liu, S., and Asada, H. H., 1997, "Modeling of Vapor Compression Cycles for Multivariable Feedback Control of HVAC Systems", ASME Journal of Dynamic Systems, Measurement & Control*,* **119**(2), pp. 183-191.

[3] Rasmussen, B. P., 2002, "Control-Oriented Modeling of Transcritical Vapor Compression Systems", M.S. Thesis, University of Illinois at Urbana-Champaign, IL.

[4] Dane, M., 2002, "Investigation of Control Strategies for Reducing Mobile Air Conditioning Power Consumption", M.S. Thesis, University of Illinois at Urbana-Champaign, IL.

[5] Forrest, W. O., Bhatti, M. S., 2002, "Energy Efficient Automotive Air Conditioning System", SAE World Congress, pp. 57-82.

[6] Magnuski, M. J., 1988, "Review of ASHRAE Symposium on the application of adaptive Control to real HVAC applications", International Journal of Refrigeration-Revue Internationale du Froid, **11**(1), pp. 11-15

[7] Singh, G., Zaheer-uddin, M., Patel, R. V., 2000, "Adaptive Control of Multivariable Thermal Processes in HVAC Systems", Energy Conservation and Management*,* **41**(15), pp. 1671-1685

[8] Ozsoy, C., 1993, "Self Tuning Control of a Heating, Ventilating and Air Conditioning System", Proceedings Institution of Mechanical Engineers, **207**, pp. 243-251

[9] Shah, R., 2003, "Dynamic Modeling and Control of Single and Multi-Evaporator Subcritical Vapor Compression Systems", M.S. Thesis, University of Illinois at Urbana-Champaign, IL.

[10] White, F. M. 1994, *Fluid Mechanics,* McGraw-Hill, Inc., New Jersey, pp. 192-214.

[11] Vapor Compression Cycle Control, http://mr-roboto.me.uiuc.edu/VC3

[12] Kokotovic, P. V., 1986, *Singular Perturbation Methods in Control: Analysis and Design,* Academic Press Inc., Orlando Florida.

[13] Skogestad, S., Postlethwaite, I., 1996, *Multivariable Feedback Control: Analysis and Design,* John Wiley & Sons, New York.

[14] Ahmed, M. S., Sait, N., 1989, "State-Space Adaptive Control through a Modified Bootstrap Algorithm for Parameter and State Estimation", IEE Proceedings, **136**, Pt. D (5), pp. 215-224.

[15] Nazaruddin, Y. Y., Unbehauen, H., 1995, "Design of a Multivariable State-Space Adaptive Controller and its Application to a Turbo-Generator Pilot Plant", IFAC Adaptive Systems in Control and Signal Processing, pp. 185-190.

[16] Astrom, K.,J., Wittenmark, Bjorn, 1995, *Adaptive Control*, Addison-Wesley Publishing Company, Inc., Massachusetts.

[17] Ljung, L., 1999, *System Identification-Theory for the User*, Prentice hall, Upper Saddle River, New Jersey.

[18] Franklin, G. F., Powell, J. D., Workman, M., 1998, *Digital Control of Dynamic Systems*, Addison-Wesley Longman Inc., California.

[19] Anderson, B. D. O., Moore, J. B., 1990, *Optimal Control*, Prentice Hall, New Jersey.